Transitional Kindergarten Chapter

of the

Mathematics Framework

for California Public Schools:
Kindergarten Through Grade Twelve

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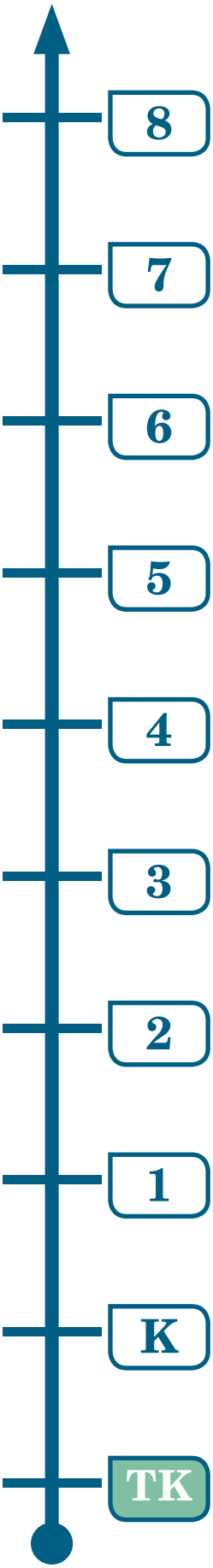


Transitional Kindergarten

The Kindergarten Readiness Act of 2010 (Senate Bill 1381, Chapter 705, Statutes of 2010) changed the entry-age requirements for kindergarten in California's public schools.[1] It also required local educational agencies to offer transitional kindergarten (TK) classes in addition to traditional kindergarten classes starting in the 2012–13 school year. Transitional kindergarten is defined in California *Education Code* section 48000(d) as "the first year of a two-year kindergarten program that uses a modified kindergarten curriculum that is age and developmentally appropriate." Traditional kindergarten, by contrast, is a one-year program with grade-specific curriculum.

The Kindergarten Readiness Act requires districts to provide students in TK programs with instruction in a modified kindergarten curriculum that is age and developmentally appropriate, but it does not specify what that curriculum should be. Districts must determine what "age and developmentally appropriate" means in terms of curriculum. The law also defines transitional kindergarten as the first year of a two-year kindergarten program, so preschool programs and TK programs within a district should be distinct. To determine the

1. California *Education Code* section 48000(a) specifies that a child shall be admitted to kindergarten at the beginning of the school year if the child will have his or her fifth birthday on or before the following dates: November 1 for the 2012–13 school year, October 1 for the 2013–14 school year, and September 1 for the 2014–15 school year.

type of modified curriculum to implement, each district needs to consider how transitional kindergarten fits into its early education system and how TK education differs from instruction in preschool and traditional kindergarten classes. Instructional leadership at both the district and school levels is necessary to ensure that TK programs meet the instructional and developmental needs of young learners. Ideally, teachers and other professionals who know mathematics content and are well versed in child development theories will develop the TK curriculum. If transitional kindergarten will truly serve as a bridge between preschool and traditional kindergarten, coordination and articulation between preschool programs (in the district and the community) and traditional kindergarten classes must occur.

Student Learning in Transitional Kindergarten

Unlike preschool or kindergarten, transitional kindergarten does not have grade-specific content standards. Therefore, the guidelines in this chapter reflect the range of abilities that students may possess in the period between preschool and kindergarten, but they are not specific to a grade-level standard. Each domain section in the chapter includes particular California Preschool Learning Foundations (for children at age 60 months) and corresponding kindergarten standards from the California Common Core State Standards for Mathematics (CA CCSSM). Sample activities that illustrate connections between the foundations and the standards are provided.

To build the foundation for success in traditional kindergarten and beyond, instructional time for mathematics in transitional kindergarten should focus on two critical areas: (1) representing, relating, and operating on whole numbers; and (2) geometry, with a focus on identifying and describing shapes and space, as well as analyzing, comparing, and composing shapes (California County Superintendents Educational Services Association [CCSESA] 2011b). To help students gain a deeper understanding of mathematics, the Standards for Mathematical Practice should be connected to content instruction.

> **"Instructional time should focus on two critical mathematical areas. One area is representing, relating, and operating on whole numbers . . . The second important area is geometry with a focus on identifying and describing shapes and space; and analyzing, comparing, creating, and comparing shapes. These two areas are intricate and complex and build the foundation for future learning in mathematics. While both prepare the young learner for more formal mathematics instruction, learning time should be devoted to number sense more than any other topic in mathematics."**
>
> **—CCSESA 2011b, 26**

It is important to remember that all students need instruction that is appropriate for their developmental level and provides opportunities for growth. In the classroom, this means that if a student is struggling with some of the California Preschool Learning Foundations, he or she should be provided with opportunities to develop abilities in those areas. Similarly, if a student meets some or all of the kindergarten standards, that student should be provided with learning opportunities that extend beyond the standards.

In transitional kindergarten, developmentally appropriate instruction involves hands-on activities for students and learning experiences in small- to medium-size groups. Particularly important are opportunities to support mathematical vocabulary acquisition in teacher–student and student–student interactions. Questions of all kinds support mathematical thinking and problem solving, especially open-ended and more challenging questions. Although the CA CCSSM do not emphasize calendar-time activities, these activities may be valuable for students' social and academic development if they support important goals such as developing social skills (group participation, taking turns, and cooperation), language skills, understanding of sequence, and number concepts.

High-quality support of mathematics education includes these important factors:

- A mathematically rich environment
- Frequent opportunities for mathematical discourse
- Engaging and meaningful mathematics activities
- Explicit instruction
- Modeling of mathematical thinking
- Nurturing of students' mathematical explorations

A *mathematically rich environment* includes a mathematics center that is refreshed on a regular basis; posters (e.g., showing shapes and numbers with sets) or wall sections devoted to interesting mathematics problems (*Are there more ducks or geese? Fewer brown birds or gray birds?*); a variety of manipulatives (teddy bears, snap-together cubes, dinosaurs, vehicles, and the like); unit blocks; shopping paraphernalia (money, cash register, labels for prices, grocery store items); two- and three-dimensional shapes; attribute blocks; and so forth.

Frequent opportunities for mathematical discourse and "math talk" build mathematical understanding and vocabulary. *Mathematical discourse* requires thinking on one's feet, knowing mathematics vocabulary, and having definitions of mathematical terms and concepts that make sense to students. The following examples of mathematical discourse are rich with mathematical vocabulary (e.g., *triangle, sides, corners, count, more, bigger, longer,* and number words for 1–7).

Example 1

Student Andrew: "Is this a triangle?" (*Holds up a square.*)

Teacher: "What do you think, children?" (*Asks other children in the small group to contribute.*)

Students, in unison: "No!"

Teacher: "Why not?"

Student Zahra: "Because a triangle doesn't have four sides."

Teacher: "That's right. How many sides does a triangle have?"

Student Alexander: "Three!"

Teacher: "How many corners does a triangle have?"

Student Alexander: "Three, just like the sides!"

Example 2

Student Nora: "Sami isn't being fair. He has more trains than I do."

Teacher: "How do you know?"

Student Nora: "His pile looks bigger!"

Student Sami: "I don't have more!"

Teacher: "How can we figure out if one of you has more?"

Student Nora: "We could count them."

Teacher: "Okay, let's have both of you count your trains."

Student Sami: "One, two, three, four, five, six, seven."

Student Nora: "One, two, three, four, five, six, seven." (*Fails to tag and count one of her eight trains.*)

Student Sami: "She skipped one! That's not fair!"

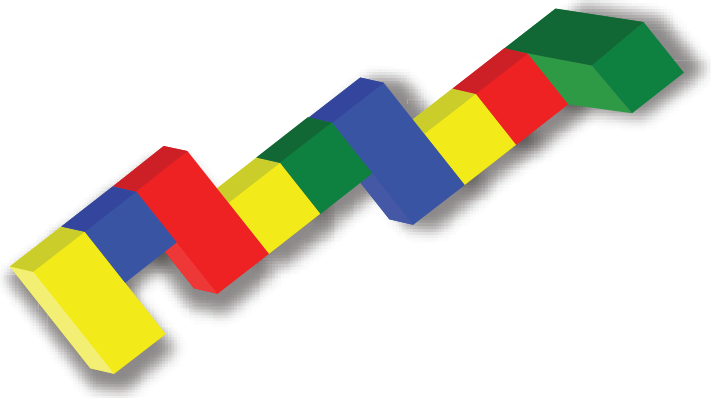
Teacher: "You are right; she did skip one. We could count again and be very careful to make sure not to skip—but can you think of another way that we can figure out if one of you has more?"

Student Sami: "We could line them up against each other and see who has a longer train."

Teacher: "Okay, show me how you do that. Sami, you line up your trains, and Nora, you line up your trains."

Engaging and meaningful mathematics activities are those that encourage students to think mathematically about the world around them. These frequently require careful planning. For a student who is interested in dinosaurs, helping him or her make a t-chart of herbivores and carnivores (using pictures or toy versions of the dinosaurs or writing the names of the dinosaurs from a book) and then having the student count the number of dinosaurs in each category may be a highly engaging activity. Some students enjoy the challenge of recreating structures with building blocks that connect or snap together or with magnetic builders (see figure TK-1). Create a structure with one of these sets, and ask a student to recreate it, including exact shapes, colors, and positions. Then ask a student to create a set that you or other students duplicate—tell the student to make it as difficult as possible. Then have them analyze whether you and the other students recreated it correctly.

Figure TK-1. A Structure Made of Building Blocks



Explicit instruction is vital in transitional kindergarten. It allows teachers to support students' acquisition of concepts that may not come up in play or other classroom activities. Explicit instruction does not mean *didactic;* rather, it means purposefully providing activities that support the understanding of a mathematical concept. The previous example of the dinosaur sorting activity is explicit (and yet highly engaging to a dinosaur fan!). Other examples of explicit instruction include dividing a set of toy trucks into three equal (and fair) shares and measuring how many children, lying end to end on the floor, it would take to equal the length of a whale shark. Then the teacher could ask the students to figure out how many mice it would take to equal the length of the shark: *Does it take more mice or more children?* All of these activities are purposeful, explicit, and contain important mathematical concepts.

Modeling of mathematical thinking provides students with strategies, techniques, and a path to deeper and more flexible understanding of mathematical concepts. There are many ways to sort students into groups—by color of clothing, laced shoes versus non-laced shoes, counting off, or by using the first or last letter of students' names. Visually and verbally modeling these sorting techniques helps students understand that there is more than one way to solve a problem. Teachers encounter mathematics problems throughout the day. Pencils are needed at each table (*How many at each table? What is the total number of pencils needed?*). More milk cartons are needed from the cafeteria (*How many more?*). Other questions arise: *How many minutes before lunch time? How many cotton balls are needed for this activity?* Solving these and other problems out loud with students allows students to see the usefulness of mathematics in real-world situations. In addition, visual aids such as a list of numbers with dots in 5-patterns above them may support analysis and learning of number words and quantities.

Finally, *nurturing of students' mathematical explorations* may create a classroom atmosphere where students believe they can solve problems and learn fun new concepts. Discovering repeating numbers in a hundreds chart is eye-opening for a young student. It can also be magical for a student to realize that 1 plus any whole number equals the next number in the counting sequence. Activities like these nurture students' interest and encourage future mathematical investigations.

Creating a learning environment that supports foundational mathematics is critical for the acquisition of later, more complex mathematical knowledge and skills. Research shows that early mathematics skills at entry to kindergarten are predictive of later academic success in both reading and mathematics (Duncan et al. 2007). Transitional kindergarten provides an excellent opportunity to continue building on students' mathematical understandings. Because children arrive at school with varied mathematical experiences, differentiated instruction is an essential part of classroom teaching; see the Universal

Access chapter for more information. Understanding each student's development and fine-tuning instruction to meet each student's needs are critical to providing quality education for all students. This is also true for English learners and students with disabilities. Although whole-group activities may be useful for introducing a concept or playing a game, smaller groups or one-on-one interactions are necessary for student acquisition of in-depth knowledge of concepts and teachers' understanding of each student's mathematical thinking, knowledge, and skills.

References to the Standards for Mathematical Practice (MP) are woven throughout the activity examples in this chapter. The MP standards describe how mathematically proficient students engage in mathematics and suggest behaviors to nurture in students. The MP standards are appropriate for transitional kindergarten students and should be integrated throughout instruction. Examples of these practices that are specific to transitional kindergarten are provided in table TK-3 at the end of the chapter.

One approach to developing a modified curriculum that is age and developmentally appropriate is to consider the intersections between the California Preschool Learning Foundations and the CA CCSSM for kindergarten (CCSESA 2011b). Transitional kindergarten may be thought of as an opportunity to introduce students to some of the kindergarten standards rather than expecting students to strive for mastery of those standards. Especially at the beginning of a TK program, a modified curriculum could provide more hands-on activities, more learning through play and exploration, and more time to develop students' mathematical skills and conceptual understandings in core lessons about smaller numbers. It should focus on developing skills and habits of mind that lead to success in traditional kindergarten, including problem solving, persistence, and reasoning.

In 2012, the California Department of Education published a document—*The Alignment of the California Preschool Learning Foundations with Key Early Education Resources* (http://www.cde.ca.gov/sp/cd/re/documents/psalignment.pdf) [CDE 2012a]—that connects the California Infant/Toddler Learning and Development Foundations, Head Start Child Development and Early Learning Framework, California Preschool Learning Foundations, and CA CCSSM. Table TK-1 shows the connection between the California Preschool Learning Foundations and the Standards for Mathematical Practice. Similarly, table TK-2 shows the alignment between the California Preschool Learning Foundations and the kindergarten CA CCSSM.

Table TK-1. Alignment Between the California Preschool Learning Foundations and the Standards for Mathematical Practice

California Preschool Learning Foundations	Standards for Mathematical Practice for Grades K–12 (MP)
Mathematical Reasoning Children use mathematical thinking to solve problems in their everyday environment.	**MP.1** Make sense of problems and persevere in solving them. **MP.2** Reason abstractly and quantitatively. **MP.3** Construct viable arguments and critique the reasoning of others. **MP.4** Model with mathematics. **MP.5** Use appropriate tools strategically. **MP.6** Attend to precision. **MP.7** Look for and make use of structure. **MP.8** Look for and express regularity in repeated reasoning.

Table TK-2. Alignment Between the California Preschool Learning Foundations and the California Common Core State Standards for Mathematics (Kindergarten)

California Preschool Learning Foundations		California Common Core State Standards—Kindergarten
Mathematics		**Mathematics**
Number Sense	→	**Counting and Cardinality**
Children understand numbers and quantities in their everyday environment.	→	Know number names and the count sequence Count to tell the number of objects Compare numbers
Children understand number relationships and operations in their everyday environment.	→	**Operations and Algebraic Thinking** Understand addition as putting together and adding to, and subtraction as taking apart and taking from **Number and Operations in Base Ten** Work with numbers 11–19 to gain foundations for place value
Algebra and Functions (Classification and Patterning)		**Measurement and Data**
Children sort and classify objects in their everyday environment. Children recognize/expand understanding of simple repeating patterns.	→	Classify objects and count the number of objects in categories
Measurement		**Measurement and Data**
Children compare, order, and measure ojects.	→	Describe and compare measurable attributes
Geometry		**Geometry**
Children identify and use shapes.	→	Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres). Analyze, compare, create, and compose shapes.
Children understand positions in space.	→	Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

Integration of Domains

The following tables integrate the California Preschool Learning Foundations for children at around 60 months of age and the corresponding kindergarten domains from the CA CCSSM. The tables are provided to facilitate district-level discussions on the development of a modified curriculum for mathematics instruction in transitional kindergarten that is age and developmentally appropriate. Each table includes these elements:

- California Preschool Learning Foundations and corresponding CA CCSSM kindergarten standards
- Vocabulary—a list of vocabulary words that students should acquire as they expand their understanding of the concepts
- What it looks like—examples of what understanding the concepts might look or sound like in the classroom (in ascending order of complexity)
- Big ideas—some of the main ideas involved in grasping the concepts related to the standard(s)
- Instructional issues—misconceptions or common conceptual difficulties that students might have
- Activities—classroom exercises that support the acquisition of the abilities embodied in the California Preschool Learning Foundations and CA CCSSM (in ascending order of complexity)

California Preschool Learning Foundations (at around 60 months of age) *Number Sense*	CA CCSSM – Kindergarten *Counting and Cardinality (CC)*
Children expand their understanding of numbers and quantities in their everyday environment. **PLF.NS–1.1** Recite numbers in order to 20 with increasing accuracy.[2]	**Know number names and the count sequence.** **K.CC.1** Count to 100 by ones and by tens. **K.CC.2** Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
Vocabulary: Number words (e.g., *one, two, three,* and so on, from 1 to 100), *count, count by, count from, number, next number, How did you figure that out?*	
What it looks like: • While playing hide-and-seek, Ezra counts to 20 before looking for the other children. • When asked to count as high as she can, Melia counts to 50. • When asked how old he is, Kenji answers, "I'm five, and then I'll be six, seven, eight, nine, ten! (**MP.7, MP.8**)	

2. In the *California Preschool Learning Foundations, Volume 1* (CDE 2008), this foundation is listed only as "1.1" in the Number Sense strand. A naming pattern was created for the mathematics framework to make clearer comparisons between the preschool foundations and California's Common Core State Standards. For **PLF.NS–1.1,** *PLF* stands for Preschool Learning Foundations, *NS* stands for Number Sense, and *1.1* is the specific foundation referenced. Also note that numerals, not the spelled-out number names that appear in the published PLF document, are used throughout the mathematics framework in places where preschool foundations are listed.

Big ideas: Students learn to recite numbers before they can apply one-to-one concepts to counting objects or understand cardinality (i.e., the last number counted represents the numerosity of the set). Encourage students to slow down as they count. After students have had experiences counting from 1, have them start counting in the middle of the counting sequence to encourage conceptual understanding of the order of numbers.

Instructional issues: An important goal in early mathematics instruction is for students to achieve fluency with the counting sequence. Students may learn a short sequence of numbers (*"four-five-six"*) and not understand that they are separate numbers (similar to the *"l-m-n-o-p"* issue when learning the alphabet). Numbers 11 through 15 may be difficult for students to learn because these numbers do not follow the pattern of 16 through 19 (the number followed by *teen*). Use discussions about how these numbers are kind of funny—calling attention to the irregularity of these number names may make it easier for students to remember that the names do not follow the regular naming pattern.

Note: Saying the counting numbers is sometimes referred to as verbal or rote counting and does not indicate an understanding of object counting with one-to-one correspondence.

Activities: Transition times are useful for providing opportunities to learn the counting numbers. Students may count how long it takes to clean up the blocks, sit in a circle, and so on. These are not precise measures of time; rather, they provide students with chances to exercise their newfound rote counting abilities. (**MP.4**)

Using a puppet named George, tell the students a story about how George has difficulty remembering how to count. Tell them that you want them to help George figure out when his counting is incorrect. Ask the students to raise their hands when they hear George make a mistake and to remember George's counting mistake. In George's voice, count to 10, skipping or repeating one number in the sequence. Call on the students who raise their hands to describe George's mistake. Ask questions to make sure students thoroughly describe the mistake and how George can fix it. (**MP.2, MP.3, MP.4, MP.6**)

During whole-group time, ask the students to sit in a circle and tell them that they are going to play a counting game. Tell them that this is a fancy game of counting called "Everybody Gets a Number." Choose a child to start the counting sequence. That child says "One" aloud; the child sitting to his or her left (going clockwise around the circle) says the next number, and the counting continues with each child in the circle. When a child says an incorrect number or does not know the next number, ask for the child to his or her right to help out. If that child does not know the number, ask the child to his or her right (keep asking the next student to the right until you find a child who can help). As students advance in knowledge, increase the difficulty of the game by asking the students to count faster, make the number goal higher, start with a number other than 1, or count by tens. (**MP.1, MP.4, MP.7**)

California Preschool Learning Foundations (at around 60 months of age) *Number Sense*	CA CCSSM – Kindergarten *Counting and Cardinality (CC)*
Children expand their understanding of numbers and quantities in their everyday environment. **PLF.NS–1.2** Recognize and know the names of some written numerals.	**Know number names and the count sequence.** **K.CC.3** Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).
Vocabulary: *Zero*	
What it looks like: • Thomas sees the numeral 4 on the wall and says, "I'm that number!" • Zeke paints the numeral 5 several times at the easel. • Using a puzzle that involves matching numbers with objects, Susan correctly matches the numerals 6 through 10 with pictures of sets of animals that number 6 through 10. (**MP.2, MP.4, MP.6**) • After drawing a pumpkin with four teeth, Maria draws a pumpkin with no teeth, laughs, and then says, "Look, *zero* teeth!" (**MP.2, MP.4**)	
Big ideas: Numerals (written or printed numbers) can describe the numerosity of a set of objects. Zero represents an empty set (in other words, no objects to count).	
Instructional issues: Students learn to count with smaller sets before they learn to count larger sets. Students may draw numbers backwards or confuse numbers that look similar to each other (e.g., 6 and 9). The concept of *zero* is difficult for students to understand and may require many examples and experiences.	
Activities: Ask students to go on a Number Hunt around the classroom. The game may be played in a variety of ways. Students could look for any numeral and then name it when called upon. Alternatively, the teacher might ask students to look for particular numerals. Number cards (one for each student in the classroom; some numbers may appear on more than one card) may be hidden around the room, and then the teacher can ask each student to find a number card and name the number on his or her card when called upon. Students create their own number cards (with the numerals 0 through 10), decorating them as they wish (using construction paper, index cards, card stock, plain white paper, and the like). The teacher then asks the students to put the cards in order and varies the activity by having students trade sets and put their new cards in order. (**MP.2, MP.4, MP.6**) Give students a number card (or let them choose) and ask them to find the same number of objects in their environment. Students bring their card and objects back to a central location (perhaps a rug or table) and share their findings with each other. With multiple students and different number cards, they can order their number cards and objects. To help students understand the concept of *zero,* hold up a number card with the numeral 0 and encourage the students to discuss how many objects they could match with the card. If appropriate, discuss with students how they know their numeral card matches the set they have displayed. (**MP.2, MP.3, MP.4, MP.6**)	

California Preschool Learning Foundations (at around 60 months of age) *Number Sense*	CA CCSSM – Kindergarten *Counting and Cardinality (CC)*
Children expand their understanding of numbers and quantities in their everyday environment. **PLF.NS–1.3** Identify, without counting, the number of objects in a collection of up to four objects (i.e., subitize).* **PLF.NS–1.4** Count up to 10 objects, using one-to-one correspondence (one object for each number word) with increasing accuracy. **PLF.NS–1.5** Understand, when counting, that the number name of the last object counted represents the total number of objects in the group (i.e., cardinality). **The Alignment of the California Preschool Learning Foundations with Key Early Education Resources* (CDE 2012a) places this foundation in a separate category that is not aligned with the CA CCSSM. It is retained here to show the connection between naming the numerosity of a set attained through subitizing and learning to count a set.	**Count to tell the number of objects.** **K.CC.4** Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. **K.CC.5** Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1 to 20, count out that many objects.
Vocabulary: *How many, one more, all together, in all, total*	
What it looks like: • Nathan glances at the number cube on the table and says, "Look! I got three!" • DeSean lines up his eight toy cars and, touching each one, counts accurately 1 through 8. (**MP.6**) • The teacher asks Talia to count how many students are in the group. Talia counts six students and then announces, "There are six." (**MP.8**) • When one more student joins the group, the teacher asks, "Now how many are in the group?" Talia answers, "That's easy—one more, that's seven!" (**MP.2, MP.8**) • Diamond is passing out pencils at each table; she accurately places six pencils on each of the four tables. (**MP.2, MP.4, MP.6, MP.8**)	
Big ideas: Students at this age can subitize (immediately, and without counting, perceive a quantity) up to about four objects. This may increase to six when the objects are in a stereotypical arrangement (e.g., six pips on a domino). Cardinality refers to the ability to determine the numerosity of a set. Students initially count each item in a set, but when asked "How many?", they will count the set again. When students gain an understanding of cardinality, they will answer with the last number that they counted instead of counting the set again and know that the last number tells how many there are.	

Instructional issues: It may take a while for students to construct strategies to keep track of what has been counted in a set. Two of these strategies are touching each object in a row until the end has been reached and moving aside the objects already counted. Counting an existing set is easier for students than creating a smaller set from a larger set (e.g., taking exactly six teddy bears from a large container of many bears) because they have to remember the number to which they are counting while counting. When beginning activities that require a particular number of objects (e.g., five cards), teachers can encourage conceptual understanding of cardinality by having the students count out their cards from the larger set of cards instead of doing it for them.

Activities: Create a Number Wall with a "Number of the Week" where students display sets of pictures (magazine, drawings, stickers, and so on) that are equal in number (e.g., six magazine pictures of trees). Arrange pictures so that they are in groups that can be subitized. Each week, change the "Number of the Week." (**MP.2, MP.4, MP.6**)

An activity that encourages both numeral recognition and object counting can be created using a numeral strip (piece of paper containing a row of boxes with numerals printed in them, beginning with the numeral 1), number cubes or a spinner, and counters. Students take turns rolling the number cubes (or spinning) and then count out that many counters (teddy bears, cars, and the like) to show they are correct; then they cross out that numeral on their numeral strip. The game ends when all students have all numerals crossed out on their numeral strips. Note that numeral strips should have all possible number cubes or spinner numerals in order. (**MP.1, MP.2, MP.4, MP.6, MP.7**)

Play board games that require students to count spaces; such games usually come with number cubes or spinners. Note that the use of a number line is not formally introduced in the CA CCSSM until grade two. All number lists or number paths should have the numbers within a shape (usually squares) that may or may not be connected to adjacent shapes. (**MP.2, MP.4, MP.6, MP.7**)

California Preschool Learning Foundations (at around 60 months of age) *Number Sense*	CA CCSSM – Kindergarten *Counting and Cardinality (CC)*
Children expand their understanding of number relationships and operations in their everyday environment. **PLF.NS–2.1** Compare, by counting or matching, two groups of up to five objects and communicate "more," "same as," or "fewer" (or "less").	**Compare numbers.** **K.CC.6** Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. **K.CC.7** Compare two numbers between 1 and 10 represented as written numerals.
Vocabulary: *More, fewer, less, same as, greater than, less than, more than*	

What it looks like:

- Jasmin and Lucas are playing in the block area, trying to divide the long blocks equally between themselves. In lining them up in one-to-one correspondence, Lucas says, "You have five and I only have four. You have more than I do!" (**MP.2, MP.3, MP.5, MP.6**)
- John and Jamal are playing with trains. After counting the trains several times, Jamal says, "I have eight trains, and you only have six! That's not fair. If I give you one of my trains, then we'll both have seven." (**MP.2, MP.3, MP.6**)
- Angel and Lisa are looking at graphs showing numbers of students who like various fruits. Angel remarks, "Look, more like apples than oranges. See? Apples has an 8 and oranges only has a 2." (**MP.2, MP.3, MP.4, MP.6, MP.8**)

Big ideas: Students may initially compare sets perceptually ("That one has more!") or by lining them up in one-to-one correspondence, or they may count both sets to compare quantitatively ("One, two, three. One, two, three—we both have three!"). Moving from comparing two sets of objects to comparing numerical symbols may be difficult for students. Encourage conceptual understanding by offering many opportunities to use numerals with matching sets of objects.

Instructional issues: Fair sharing in the classroom can play a big part in providing opportunities for students to compare quantities (everyone wants a fair share!). Some students may be confused by the length or size of a set of objects when comparing it to another set. Lining up objects in one-to-one correspondence may help students ascertain whether one set is larger than another. Students may struggle to understand that the numerosity of a set does not change if nothing is added or taken away. Using the question "Is this group really more, or does it just look like more?" may be helpful. Teachers should use the words *fewer* and *less* more frequently than the word *more,* because children typically have fewer opportunities to learn the words *fewer* and *less*.

Activities: Card game of Compare (comparing numerals or sets of icons on cards). Each student receives a set of cards with numerals or sets of objects on them (within 5). Working with a partner, each student flips over one card (like the card game "War"). The students decide which card represents *more* or *fewer,* or if the cards are *the same as.* (**MP.2**)

Play a game in which you create a set of counters (1–9). Count the counters with a small group of students. Then either add one more or take away one counter from the set. Then ask the students to figure out how many there are in the set. Involve the students in a discussion of how they can figure out the answer (there are several ways) and how they know the answer; involve the entire group in the discussion, making sure that all students participate. To support students' understanding, display a number list or path in the classroom—at the students' eye level—showing the numerals in order from 1 (these are the counting numbers) with dots in groups of 5 above them. (**MP.1, MP.2, MP.3, MP.4, MP.8**)

In a small group, play a game with counters (teddy bears, cars, and the like) with the students. Create a set and ask the students to create their own sets of the same number. Work with 1 to 10 counters. To increase the complexity of the game, ask the students to create a set that is one more or one less than your set, or have them create a set that has more (or fewer) objects than yours. Have a discussion with the students, asking each child to describe her or his set—whether it is larger or smaller than yours and how they know. If students do not use the phrase "the extra" during the discussion, pointing to the objects that are "the extra" may be helpful. The group with more has extra objects. (**MP.1, MP.2, MP.3, MP.4, MP.7, MP.8**)

California Preschool Learning Foundations (at around 60 months of age) *Number Sense*	CA CCSSM – Kindergarten *Operations and Algebraic Thinking (OA)*
Children expand their understanding of number relationships and operations in their everyday environment. **PLF.NS–2.2** Understand that adding one or taking away one changes the number in a small group of objects by exactly one. **PLF.NS–2.3** Understand that putting two groups of objects together will make a bigger group and that a group of objects can be taken apart into smaller groups. **PLF.NS–2.4** Solve simple addition and subtraction problems with a small number of objects (sums up to 10), usually by counting.	**Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.** **K.OA.1** Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. **K.OA.2** Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. **K.OA.3** Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). **K.OA.4** For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. **K.OA.5** Fluently add and subtract within 5.
Vocabulary: *Bigger, smaller, add, subtract, take away, addition, subtraction, adding, subtracting, make 10, all together, equals, the same as, in all, total, amount left*	
What it looks like: • Tony announces, "Look, if we put your blocks with my blocks, we have a bigger pile! We have more." (This is an example of an Add To/Result Unknown situation. See table GL-4 in the glossary.) (**MP.8**) • Miriam says, "I have three cows and two pigs. That makes one, two, three, four, five. Five animals!" (This is an example of a Put Together/Total Unknown addition situation. See table GL-4 in the glossary.) (**MP.2, MP.4, MP.6**) • While playing in the block area, José says to Antonio, "If we put your cylinders with my cylinders, we'll have, one, two three, four, five, six cylinders—enough for the factory smokestacks!" (This is another example of a Put Together/Total Unknown addition situation.) (**MP.2, MP.4**) • Oscar says, "There are five cars, but two are broken, so we can only use three of them." (This is an example of a Take From/Results Unknown subtraction situation. See table GL-4 in the glossary.) (**MP.2, MP.4**)	

Big ideas: Most young students use a counting-all strategy to solve addition problems with objects. That is, they count all of the objects in both sets. However, some students will go on to learn the more advanced grade-one strategy of counting on from the larger set (e.g., when adding four and two objects, they begin with "Four" and continue, "five, six"). Provide students with opportunities to take apart groups of objects and examine how many they started with, how many were taken away, and how many are left. Table GL-4 in the glossary illustrates the variety of addition and subtraction situations and difficulty level. Students in transitional kindergarten may work with problems involving *Add To/Take From with Result Unknown, Put Together/Take Apart with Total Unknown, and Both Addends Unknown.*

Instructional issues: Students can directly model addition and subtraction situations given by the teacher or taken from their own lives. Provide frequent opportunities to engage in addition and subtraction activities involving story situations; students should learn to tell such stories and not just solve them. Initially working within addends less than 5 encourages in-depth understanding of addition and subtraction concepts. Encourage problem solving through the use of fingers, drawings, and manipulatives. When introducing the equal sign (=), emphasize and illustrate that the symbol means equal (not "the answer is"). Stress that the *quantities* represented on the left and right sides of this symbol must be the same (they can be objects, numerals, or expressions). Use the equation form $5 = 3 + 2$ when taking apart a number to show both addends. Using the word *partners* for addends helps students to conceptualize these numbers as hiding inside a number.

Activities: While reading books, ask questions about numbers. For instance, in a book about dogs, on the page showing a picture of two dogs, ask how many dogs there are, and then ask questions such as these: *How many legs does one dog have? How many legs do two dogs have? If one dog left the page, how many legs would be left?* (**MP.1, MP.2, MP.4, MP.6, MP.7**)

During small-group or whole-group time, have students represent with their fingers the addends in a story problem. Call on individual students to explain how they decided how many fingers to choose for each hand. *Example:* "One day, two baby dinosaurs hatched out of their eggs. The mama triceratops was so excited that she called to her auntie to come and see. Then four more baby dinosaurs hatched! How many dinosaurs hatched all together? Mirasol, can you show me how many fingers you used?" Note that children from different cultures learn to show numbers on their fingers in different ways. Children may start with the thumb, the little finger, or the pointing finger. Support all of these ways of showing numbers with fingers. (**MP.2, MP.4, MP.5**)

Present students with story problems and encourage the students to solve the problems with manipulatives or drawings. Initially, talking about how one can represent the problem on paper or with manipulatives might be useful. *Example:* "Four cars are waiting to be repaired at the repair shop. These blocks will be the cars [puts four blocks in front of the students]. Paul said that his garage has only two car lifts. If we put these two cars up on the lifts [moves the blocks away from the group of four], how many cars are waiting for their turn on the lifts?" (**MP.2, MP.4, MP.5**)

California Preschool Learning Foundations (at around 60 months of age) *Measurement*	CA CCSSM – Kindergarten *Measurement and Data (MD)*
Children expand their understanding of comparing, ordering, and measuring objects. **PLF.M–1.1** Compare two objects by length, weight, or capacity directly (e.g., putting objects side by side) or indirectly (e.g., using a third object). **PLF.M–1.2** Order four or more objects by size. **PLF.M–1.3** Measure length using multiple duplicates of the same-size concrete units laid end to end.	**Describe and compare measurable attributes.** **K.MD.1** Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. **K.MD.2** Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*
Vocabulary: *Longest, shortest, largest, smallest, heaviest, lightest, highest, lowest, most, least, more than, less than, same as, shorter than, longer than, larger than, smaller than, heavier than, lighter than*	
What it looks like: • Jake lines up the four twigs he found on the playground in order of length. (**MP.4, MP.7**) • Tyrone pulls his train up beside Malik's with the engines lined up and says, "My train is longer than your train." (**MP.4, MP.6, MP.7**) • Dylan and Kiara are comparing the pumpkins they have drawn and cut out. Kiara puts Dylan's pumpkin on top of hers and says, "My pumpkin is bigger than yours." Kiara then measures her pumpkin's mouth with her index finger and compares it to the mouth on Dylan's pumpkin. She says, "Your pumpkin has a longer mouth than mine!" (**MP.4, MP.5**)	
Big ideas: For students in later grades, measuring usually means assigning a numerical quantity to an object (e.g., 4 pounds or 6 inches). Generally, this is referred to as *formal measurement.* However, for younger students, directly comparing these attributes *(informal measurement)* forms an important foundation for later understanding. Using duplicates of concrete objects that are the same size prepares students for thinking about repeating units (such as inches on a ruler or measuring tape). Emphasizing that these units are all the same size is an important concept that provides scaffolding for later formal measurement.	
Instructional issues: Students may not understand that in order to compare the length or height of objects, all of the objects must have the same starting point (e.g., in measuring the height of four objects, they are all placed upright on a table). Help students develop this ability. For vertical measurements, use the table or floor as the starting point. For horizontal measurement, mark the starting point with tape (by drawing a line) or with a straight stick. When students are measuring with non-standard units of measure, encourage them to use the same unit to measure each item.	

Activities: Make balance scales available in your classroom. Encourage students to use them to compare the weight of objects. Before students weigh two different objects, ask them to estimate which object is heavier and then check their guesses after the items are weighed. (**MP.5, MP.6**)

Divide the class into four to six groups of students. Choose items from your classroom with measurable attributes—for example, pencils, dolls, or trucks for length; pumpkins, balls, or beanbags for weight. Provide each group of students with one of these items. After distributing, ask each group to find objects that are shorter/longer or heavier/lighter than the item you have given them (one item per student). Give the groups about five to 10 minutes to complete this task. Then ask each group to make two piles of objects they have collected—one that consists of longer/heavier objects than the initial object you have provided and one that is shorter/lighter. The goal is for all students in the group to agree on which pile each item belongs in. Then have all students in the classroom listen as each group reports to the whole group on their decisions. In order to keep all students engaged, tell them to listen carefully to the decisions made by other groups, to be prepared to say whether they agree or not, and to explain how they might check the accuracy of the decisions. (**MP.3, MP.4, MP.8**)

Young students are fascinated by large creatures. Find a children's book about dinosaurs or elephants that talks about the size of these animals. Create an activity in which the dinosaur or elephant is drawn to scale on the playground. Ask the students how they could measure the animal with their bodies ("How many children tall is a dinosaur? How many hands tall?"). Have a discussion about whether all children are the same height or not, and ask if it would it make a difference if you measured a dinosaur with different-sized children ("Would it take fewer third-grade students laid end to end than it would if transitional kindergarten students were laid end to end?"). Be sure to include discussion of the ability or inability to measure exactly the number of children (for instance, if it takes four children and a part of a child). This is not to teach fractions, but to highlight the unit of measurement and underscore the importance of using a standardized unit (e.g., "four Sams high" is different from "four Susies high"). (**MP.2, MP.3, MP.4, MP.8**)

California Preschool Learning Foundations (at around 60 months of age) *Algebra and Functions*	CA CCSSM – Kindergarten *Measurement and Data (MD)*
Children expand their understanding of sorting and classifying objects in their everyday environment. **PLF.AF–1.1** Sort and classify objects by *one or more* attributes, into two or more groups, with increasing accuracy (e.g., may sort first by one attribute and then by another attribute).	**Classify objects and count the number of objects in each category.** **K.MD.3** Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.
Vocabulary: *Sort, group, same, different*	
What it looks like: • Jane is playing with the teddy-bear counters. The counters come in three sizes and four colors. First she divides them by color into four groups. Then she says, "Now I'm gonna put all the Daddies together and the Mamas together and the babies together." She sorts all of the bears into these three sizes. (**MP.1, MP.3, MP.4, MP.6, MP.8**) • Garrett is sorting buttons. First he sorts by color, then by size. (**MP.4, MP.6, MP.8**) • Demetrius is sorting the trains into engines, coal carriers, and flat train cars. He announces, "I have more flat cars than coal carriers, and I only have two engines." (**MP.2, MP.4, MP.6**)	
Big ideas: Objects may be sorted by more than one attribute.	
Instructional issues: Being able to sort a group of objects by more than one attribute is an important ability. Help students develop this ability by encouraging this activity in a variety of settings, not just with manipulatives. If you go on a walk, ask the students to think about how many ways trees could be grouped (e.g., by leaf shape, trunk color, type of fruit, and so forth). While eating lunch, ask students to give different ways for grouping vegetables (by color, softness or hardness, and so on).	
Activities: Attribute blocks provide a good way to encourage students to think of different ways to categorize objects. They may be grouped by color, shape, size, and thickness. (**MP.1, MP.4, MP.8**) Have students sit in four to six small groups. Have each group gather 10 to 12 objects from the classroom, and ask the students to figure out different ways to sort the items. Encourage discussion within each group as part of the decision-making process. Then have each group present its categories to the class and explain why the objects belong in the category. (**MP.1, MP.3, MP.4, MP.8**) Place 10 objects in two groups; items in each group should be related to one another in some way. Ask students, "How are these objects grouped together? Why were these objects placed in the same group? Why are there two groups of objects?" Encourage students to discuss the attributes they notice for each group and to explain their reasoning. This activity could be simple or complex, depending on the readiness of the students. (**MP.1, MP.3, MP.6, MP.7**)	

California Preschool Learning Foundations (at around 60 months of age) *Geometry*	CA CCSSM – Kindergarten *Geometry (G)*
Children identify and use a variety of shapes in their everyday environment. **PLF.G–1.1** Identify, describe, and construct a variety of different shapes, including variations of a circle, triangle, rectangle, square, and other shapes. **PLF.G–1.2** Combine different shapes to create a picture or design. **PLF.G–2.1** Identify positions of objects and people in space, including in/on/under, up/down, inside/outside, beside/between, and in front/behind.	**Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).** **K.G.1** Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to.* **K.G.2** Correctly name shapes regardless of their orientations or overall size. **K.G.3** Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid"). **Analyze, compare, create, and compose shapes.** **K.G.4** Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). **K.G.5** Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. **K.G.6** Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*
Vocabulary: *In, on, under, up, down, inside, outside, beside, between, in front, behind, below, next to, flat, solid, square, circle, triangle, rectangle, hexagon, cube, cone, cylinder, sphere, side, corner, vertex, vertices*	
What it looks like: • Xavier says, "Look, the window is a rectangle, and it has rectangles in it!" (**MP.7**) • In a class discussion about shapes, Veronica says, "A sphere is just like a ball— round all around!" (**MP.2**) • In a discussion about the prepositions *above* and *below,* Cho says, "That's funny, things can be both! Everything is above the floor and below the ceiling!" (**MP.2, MP.3**)	
Big ideas: Shapes have fixed attributes, such as the number of sides and corners. Knowledge of three-dimensional shapes is important; do not limit exposure to two-dimensional shapes. Two or more shapes can be put together to make new shapes.	

Instructional issues: Shapes should be provided in all orientations and all permutations (long rectangles, triangles with a vertex pointing down, isosceles triangles, scalene triangles), and these should be discussed to help students focus on the central attributes. Help students to understand the difference between actual representations of shapes and common objects with similar characteristics (e.g., an apple has round characteristics, but it is not a sphere). Help students begin to understand that some shapes are special cases of a larger shape category (e.g., a square is a special rectangle that has all sides of equal lengths). Students should compose and decompose shapes with right angles and not just pattern blocks made from equilateral triangles.

Activities: In a whole group or small group, talk about words that describe where something is. Examples are *in, on, under, up, down, inside, outside, beside, between, in front, behind, below,* and *next to.* Ask each student to find an example of these positions/prepositions in the classroom. One example is *over:* Ask students to find things that are over something else. Give students about five minutes to find examples. Go around the room and ask each student what the object is and what it is over (e.g., *"The exit sign is over the door"*). (**MP.1, MP.4**)

Provide opportunities for sorting by shapes. For students who are just learning about shapes, a shape sorter —a container with different-shaped openings through which corresponding three-dimensional pieces (typically made of plastic or wood) can be pushed—may be useful. Pattern blocks and attribute blocks are also useful for sorting. (**MP.4**)

Provide an activity center where students create and work with shapes. This center might include shape magnets, clay balls and toothpicks, chopsticks, paper, pencils, and scissors. Encourage students to talk about what they are creating. Provide tangram sets with pictures to compose and parquet blocks for creating designs. (**MP.4, MP.5, MP.7**)

Gather a collection of two- and three-dimensional shapes. In a whole or small group, ask students to describe the shapes one by one. For instance, hold up a triangle and ask the students to describe it. At first, students might need help learning the vocabulary words listed above. To prompt students' descriptions, ask them how many sides, corners, vertices, or faces they see when looking at a particular shape. After students are comfortable providing these sorts of descriptions, change the activity by describing a hidden shape and asking the students to guess which shape it is. (**MP.3, MP.4, MP.6, MP.7, MP.8**)

Table TK-3. Standards for Mathematical Practice—Explanation and Examples for Transitional Kindergarten

Standards for Mathematical Practice	Explanation and Examples
MP.1 Make sense of problems and persevere in solving them.	Transitional kindergarten provides an opportunity for teachers to instill the joy of mathematical problem solving. Mathematical activities should be both meaningful and challenging. Some of these activities are games (e.g., board games, card-number games, dominoes) that are useful because mathematics is being employed to solve problems. Consider using games in which no one "wins" until every student has finished, as well as games that require collaboration. Encourage students to persevere in solving problems; students often find that problems requiring a bit of time to solve can be the most rewarding. Possible prompts: *How do you know? What do you know about _____ ? What would happen if _____ ?*
MP.2 Reason abstractly and quantitatively.	Counting things for a reason—or just to get better at it—is important. Young students love to count things and to practice the counting sequence. Competence is the motivation. Many experiences in the manipulative-centered activities of transitional kindergartners are natural environments that require quantitative reasoning. Fair sharing, in particular, promotes this sort of thinking in the classroom. As students become more familiar with quantitative reasoning with objects, they become more able to reason abstractly—for example, "You have five trucks and I have four trucks, and since five is more than four, you have more trucks than I do. That's not fair!" Possible prompts: *What do you know about the number _____ ? Let's make a story about these numbers.*
MP.3 Construct viable arguments and critique the reasoning of others.	Young students are very capable of stating a point of view and defending it. Help students transfer these abilities to the domain of mathematics. Ask students how they arrived at their answer, and have them discuss with others not only the correct answer, but also the strategies used to find the answer. There are many problems with more than one correct answer (e.g., "What number is greater than five?") and more than one strategy for finding a correct answer. Model how to explain answers and discuss other solutions with classmates. Possible prompts: *How did you figure that out? What do you think about _____ ?*
MP.4 Model with mathematics.	*Modeling with mathematics* means that teachers provide models (solving a problem aloud and with manipulatives) and that students use objects to demonstrate their thinking. Possible prompt: *What could we use to _____ ?* Solve mathematical problems aloud. For example, divide a box of pencils so that each table receives one pencil for each student seated: "Let's see, there are four of you here, so we will need four pencils. One, two, three, four." Encourage students to use manipulatives to show their thinking ("Mica, can you show me how you know you shared these eight trucks fairly with Charlie?").
MP.5 Use appropriate tools strategically.	The transitional kindergarten classroom is filled with tools. These include instruments such as balance scales and measuring tapes, as well as the manipulatives and objects that students and teachers use to model mathematics. Students should have frequent opportunities to ponder which of these is appropriate to the task at hand. Possible prompts: *What could you use to help you with _____ ? How could you use a _____ to help you with _____ ?*

Table TK-3 *(continued)*

Standards for Mathematical Practice	Explanation and Examples
MP.6 Attend to precision.	*Precision* entails more than arriving at a correct answer. It also involves being able to describe strategies, arguments, and decisions with increasing skill. Descriptions become more and more precise. Triangle descriptions change from "Because it looks like a triangle" to "It has three sides and three corners." Students learn that if they do not provide accurate representations during problem solving (e.g., when drawing 3 + 5 they draw only two and five objects), then they will have problems determining accurate answers. There is a beauty in precision, and many students are entranced by this beauty (e.g., 2 + 3 is always 5!). Possible prompts: *What do you know about _____ ? What else do you notice?*
MP.7 Look for and make use of structure.	Students in transitional kindergarten will begin to see patterns as they gain experience in mathematics. For instance, 1 plus any number will always equal the next [whole] number in the sequence. Possible prompts: *What do you notice about _____ ? How is this the same as _____ ? What are two different ways we can look at these objects? Tell me about your pattern.*
MP.8 Look for and express regularity in repeated reasoning.	Young students delight in finding patterns. For example, to solve addition problems, one can always count all the objects in both sets. One can also count on from the larger set. In number decomposition, students may find (especially if they record the addends) that if the first addend is decreased by 1, then the second is increased by 1 (3 + 7 = 10; 2 + 8 = 10; 5 = 3 + 2; 5 = 2 + 3). Asking questions of students that help them examine the strategies with which they solve problems will help them see regularity in the way they solve these problems. Possible prompt: *What do you notice?*